Ad Hoc Networks 9 (2011) 16–27

Contents lists available at ScienceDirect

Ad Hoc Networks

journal homepage: www.elsevier.com/locate/adhoc

Optimal physical carrier sense in wireless networks

Kyung-Joon Park [a,*,1], Jihyuk Choi [b], Jennifer C. Hou [a,2], Yih-Chun Hu [b], Hyuk Lim [c,d]

[a] Department of Computer Science, University of Illinois, 201 N. Goodwin Avenue, Urbana, IL 61801, USA
[b] Department of Electrical and Computer Engineering, University of Illinois, 1406 W. Green Street, Urbana, IL 61801, USA
[c] Department of Information and Communications, GIST, Republic of Korea
[d] Department of Nanobio Materials and Electronics, GIST, Republic of Korea

ARTICLE INFO

Article history:
Received 20 January 2010
Received in revised form 23 March 2010
Accepted 23 April 2010
Available online 28 April 2010

Keywords:
Carrier Sense Multiple Access
Spatial reuse
Hidden node problem
Exposed node problem

ABSTRACT

We investigate the problem of maximizing Medium Access Control (MAC) throughput in Carrier Sense Multiple Access (CSMA) wireless networks. By explicitly incorporating the carrier sense threshold and the transmit power into our analysis, we derive an analytical relation between MAC throughput and system parameters. In homogeneous networks, we derive the optimal carrier sense range at a given node density as a function of the ratio between the transmit power and the carrier sense threshold. The obtained optimal carrier sense range is smaller than that for covering the entire interference range, which is in sharp contrast to what has been considered to be optimal in previous studies. Only when the node density goes to infinity, the optimal carrier sense range converges to that for exactly covering the interference range, thereby eliminating all the hidden nodes. For nonhomogeneous networks, any distributed algorithm for tuning the carrier sense threshold, in which each node tries to maximize its own throughput without coordination, may significantly degrade MAC throughput. In order to properly design a distributed algorithm, each node not only considers its own throughput, but also needs to take account of its adverse impact on others. Our analysis is verified by simulation studies under various network scenarios.

1. Introduction

Multi-hop wireless networks, e.g., wireless mesh networks, have emerged as a promising, cost-effective technology for next-generation wireless networking [1]. Their main advantage is the capability of building networks without a pre-installed infrastructure. Instead of coordinating the radio channel by a central entity, a distributed access mechanism is deployed at each node to arbitrate access to the channel. Caused by this convenience, most deployed multi-hop wireless networks are employing Carrier Sense Multiple Access (CSMA). Here, our main focus is on CSMA wireless networks.

A critical performance metric in CSMA wireless networks is network capacity, i.e., the average number of data bits that can be transported simultaneously in the network. This metric heavily depends on the level of *spatial reuse* characterized by carrier sense. For example, IEEE 802.11 Distributed Coordination Function (DCF) [2] has employed two types of carrier sense: mandatory *physical carrier sense* that monitors the signal strength of the channel, and optional virtual carrier sense that uses the Request-To-Send/Clear-To-Send (RTS/CTS) handshake to reserve the medium prior to transmission. In this paper, we mainly focus on physical carrier sense and will discuss the effect of RTS/CTS as an extension. For physical carrier sense, before each transmission, a sender listens to the channel and determines whether or not the channel is

* Corresponding author. Tel.: +1 217 419 6556.
E-mail addresses: kjp@snu.ac.kr (K.-J. Park), jchoi43@illinois.edu (J. Choi), yihchun@illinois.edu (Y.-C. Hu), hlim@gist.ac.kr (H. Lim).

[1] K.-J. Park was with the Department of Computer Science, University of Illinois at Urbana-Champaign at the time of this work. He is now with Seoul National University, Seoul, Korea.

[2] J. C. Hou was with the Department of Computer Science, University of Illinois at Urbana-Champaign at the time of this work. She is now deceased.

1570-8705/$ - see front matter © 2010 Elsevier B.V. All rights reserved.
doi:10.1016/j.adhoc.2010.04.006

busy by comparing the received signal strength with the carrier sense threshold. If the signal strength is below the carrier sense threshold, the sender considers the channel to be idle and starts its transmission. Otherwise, the sender considers the channel to be busy and defers its transmission. Since the received signal strength is proportional to the transmit power of the corresponding sender, both the carrier sense threshold and the transmit power are major control knobs for physical carrier sense.

There have been a number of studies that focus on the impact of physical carrier sense on network capacity. (We will give a detailed summary of existing work in Section 2.) Most research efforts [3–5], however, have concentrated either on the relation between physical carrier sense and Shannon capacity, i.e., the achievable channel rate under the additive white Gaussian noise channel model (instead of Medium Access Control (MAC) throughput, i.e., the average rate of successful message delivery in the MAC layer) or on the derivation of a simple condition for eliminating all the hidden nodes (without considering sufficient details of how physical carrier sense operates) [6,7]. What has not been fully investigated is the analytical relation between physical carrier sense and the MAC throughput. Hereafter, we interchangeably use network MAC throughput (aggregate MAC throughput over every node) and network capacity to denote MAC-level throughput under the saturation condition [8].

In this paper, we are interested in seeking solutions to the following questions: What is the analytical relation between network capacity and system parameters such as the carrier sense threshold and the transmit power? Is eliminating all the hidden nodes really optimal in terms of network capacity? If not, what is the optimal condition? As in the case of using Shannon capacity [3–5] to characterize network capacity, can we still quantify network capacity as a function of the ratio of the carrier sense threshold to the transmit power? Furthermore, is there any advantage of deploying nonhomogeneous networks (where the system parameters can be adjusted independently by each node) over homogeneous networks (where system parameters are set to the same values for every node)? We aim to answer the above questions in an analytical framework. Specifically, our contributions are as follows:

- By explicitly incorporating the *carrier sense threshold* and the *transmit power* into our analysis, we establish an analytical relation between network capacity and the level of spatial reuse characterized by physical carrier sense. Although there have been considerable research efforts on modeling the performance of CSMA wireless networks [8–11], none of them have explicitly incorporated the carrier sense threshold and the transmit power in their models.
- In the case of homogeneous networks, we show that network capacity depends on the ratio of the transmit power to the carrier sense threshold. By using the notion of the carrier sense range, we derive the optimal carrier sense range as an explicit function of system parameters such as node density, channel access probability, and duration of each channel state. We identify that the optimal carrier sense range is smaller than the value for exactly covering the entire interference range of the receiver, which implies that the hidden nodes will not be totally eliminated. This result is in sharp contrast to what has been considered to be optimal in previous studies. Only when the node density goes to infinity, the optimal carrier sense range converges to that for exactly covering the entire interference range, thereby eliminating all the hidden nodes. This analysis quantifies the intuitive tradeoff between the hidden node problem and the exposed node problem.
- In the case of nonhomogeneous networks, the carrier sense threshold and the transmit power should be considered independently in order to determine network capacity. The problem of maximizing network capacity in a fully distributed manner is shown to be a non-cooperative game [12]. Any selfish distributed algorithm in which every node tunes its own parameters for maximizing its own throughput, without coordination with other nodes, will fail to maximize network capacity and may result in a poor system performance. Consequently, each node needs to consider not only its own throughput, but also needs to introduce a certain form of penalty as a price for the adverse impact on others.

The remainder of the paper is organized as follows: In Section 2, we give a detailed summary of related work and highlight the difference between prior work and ours. In Section 3, we introduce the propagation and interference models used in our analysis. Then, by focusing on physical carrier sense, we characterize the MAC throughput. In Section 4, we derive an analytical relation between network capacity and system parameters. Based on the relation, we find the optimal carrier sense range, which maximizes network capacity. Then, we discuss several related issues such as the effect of RTS/CTS on network capacity and multiple data rates. We present simulation results in Section 5, and conclude the paper in Section 6 with a list of research avenues for future work.

2. Related work

We categorize related work into the following three cases.

2.1. Performance analysis of IEEE 802.11 DCF

There have been considerable studies on the performance of IEEE 802.11 DCF both in single-cell scenarios [8–10] and multi-hop networks [11]. In [8], Bianchi modeled the behavior of the binary back-off counter at one tagged node as a two-dimensional Markov chain, and derived a fixed-point model for IEEE 802.11 DCF. Calì et al. [9] derived a throughput bound by approximating IEEE 802.11 DCF with a p-persistent model. Kumar et al. [10] generalized the fixed-point analysis of Bianchi's model. Recently, Medepalli and Tobagi [11] extended Bianchi's work, and provided an analytical model that captures several important performance metrics such as throughput, delay, and fairness. In all of these previous studies, the impact of the carrier sense threshold and the transmit power has *not been fully investigated*.

2.2. Studies on physical carrier sense for improving the level of spatial reuse

Recently, a number of studies have been carried out to study how physical carrier sense affects spatial reuse [6,7]. Given a predetermined transmission rate, Zhu et al. [6] derived conditions for the carrier sense threshold in order to cover the entire interference range. Zhu et al. also proposed in [7] a dynamic algorithm for adjusting the carrier sense threshold. There have been also a number of studies on the relation between physical carrier sense and Shannon capacity [3–5]. Yang and Vaidya [4] are perhaps the first to address the impact of physical carrier sense on Shannon capacity of wireless ad hoc networks while taking into account the MAC layer overhead. Zhai and Fang [5] investigated the impact of physical carrier sense in multi-rate and multi-hop scenarios. Kim et al. [3] showed that with a scheduling-based MAC, spatial reuse only depends on the ratio of the transmit power to the carrier sense threshold. In addition, an implementation-oriented study was presented in [13]. Recently, Zhu et al. [14] derived an implicit relation for obtaining the optimal carrier sense range in homogeneous networks. A more comprehensive study on optimizing CSMA networks has been performed in [15]. In addition, a non-cooperative game-theoretic approach for control of the carrier sense threshold has been proposed in [16].

However, none of these studies have derived an *explicit* functional relation for the optimal physical carrier sense. Once we obtain a functional relation between the optimal carrier sense threshold and system parameters (node density, channel access probability, duration of each channel state, etc.), we can exploit this relation to further improve the network performance.

2.3. Transmit power control

The issue of transmit power control has been extensively studied in the context of topology maintenance [17–19], where the main objective was to preserve network connectivity while mitigating MAC-level interference and reducing power consumption. The problem of how to control the carrier sense threshold in topology-controlled wireless networks has been investigated in [20]. Use of transmit power control for maximizing capacity has been considered, for example, in [21], in which Monks et al. proposed a power control protocol, called Power Controlled Multiple Access (PCMA), where the receiver advertises its interference margin that it can tolerate on an out-of-band channel and the transmitter selects its power in order not to disrupt any ongoing transmissions. Recently, the problem of joint optimization of transmit power and carrier sense has been also proposed in [22].

3. Network model

In this section, we introduce the network model used in our analysis.

3.1. Wireless channel model and related notions

Consider a wireless network consisting of a set of N nodes, indexed from 1 to N, denoted by $\mathcal{N}$. For a given node $i \in \mathcal{N}$, let $r(i) \in \mathcal{N}$ denote the corresponding receiver of node i. Let P_i denote the transmit power of node i, g the antenna gain, and θ the path loss exponent (which typically ranges between 2 and 4), then the received power is denoted by $P_{r(i)} = gP_i d_{i,r(i)}^{-\theta}$, where $d_{i,j}$ denotes the distance between node i and node j. As a necessary condition for the receiver $r(i)$ to correctly decode the symbols, $P_{r(i)}$ should be larger than or equal to the *receive threshold* of $r(i)$, denoted by $\gamma_{r(i)}$, i.e.,

$$P_{r(i)} \geqslant \gamma_{r(i)}. \tag{1}$$

By (1), the transmission range $d_T(i, r(i))$, which is the maximum of $d_{i,r(i)}$ satisfying (1), can be obtained as $d_T(i, r(i)) = (gP_i/\gamma_{r(i)})^{\frac{1}{\theta}}$. In addition to (1), the received power $P_{r(i)}$ should be large enough so that the interference from other nodes does not prevent the receiver from correctly decoding the symbols. This condition can be usually expressed with the signal to noise interference ratio (SINR) as follows:

$$\mathrm{SINR} = \frac{P_{r(i)}}{N_{r(i)} + \sum_{j \neq i} gP_j d_{j,r(i)}^{-\theta}} \geqslant \beta_{r(i)},$$

where $N_{r(i)}$ is the ambient noise and $\beta_{r(i)}$ is called the SINR threshold of the receiver $r(i)$.

Now the *interference set* of receiver $r(i)$, denoted by $I_{r(i)}$, is defined as the set of nodes whose simultaneous transmission with node i, taken one at a time, will cause collision at $r(i)$. With negligible noise $N_{r(i)}$, $I_{r(i)}$ can be expressed as

$$I_{r(i)} = \left\{ j | P_{r(i)} / \left(gP_j d_{j,r(i)}^{-\theta} \right) < \beta_{r(i)} \right\} = \{ j | d_{j,r(i)} < d_I(i,j) \},$$

where $d_I(i,j) := (P_j \beta_i / P_i)^{\frac{1}{\theta}} d_{i,r(i)}$ is termed as the interference range. With (1) and the notion of $I_{r(i)}$, we assume that a transmission between node i and the corresponding receiver $r(i)$ is successful if $r(i)$ is inside the transmission range of i and no node in $I_{r(i)}$ is simultaneously transmitting, i.e.,

$$d_{i,r(i)} < d_T(i, r(i)) \quad \text{and} \quad d_{j,r(i)} \geqslant d_I(i,j), \tag{2}$$

for every node $j \neq i$ that is simultaneously transmitting. Note that the condition (2) corresponds to the well-known protocol model [23].

Let x_i denote the carrier sense threshold of node i. If the sensed signal level at node i is larger (smaller) than x_i, the channel will be considered busy (idle) by node i. For a given node i, let C_i denote the *carrier sense set* of node i, which is defined as

$$C_i = \left\{ j | gP_j d_{i,j}^{-\theta} \geqslant x_i \right\} = \{ j | d_{i,j} \leqslant d_C(i,j) \}, \tag{3}$$

where $d_C(i,j) := (gP_j/x_i)^{\frac{1}{\theta}}$ is the carrier sense range. Hence, node i will be silenced if any node in C_i is transmitting. In a similar manner, let L_i denote the *silence set* of node i, which is defined as

$$L_i = \left\{ j | gP_i d_{i,j}^{-\theta} \geqslant x_j \right\} = \{ j | d_{i,j} \leqslant d_L(i,j) \}, \tag{4}$$

where $d_L(i,j) := (gP_i/x_j)^{\frac{1}{\theta}}$ is the silenced range. Thus, every node $j \in L_i$ will be silenced when node i transmits. Note that $C_i = L_i$ only in a homogeneous network under a symmetric topology, and $C_i \neq L_i$ in general. Finally, H_i denote the set of hidden nodes of node i. Then, we have $H_i = H_i^- \cup H_i^+$, where $H_i^- = I_{r(i)} \setminus C_i$ and $H_i^+ = I_{r(i)} \setminus L_i$, which can be described as follows: The transmission attempt of node i will fail either if there is any ongoing transmission in H_i^-, or if any nodes in H_i^+ transmit during the transmission of node i. This is the well-known *hidden node problem*. One may partially resolve the hidden node problem by expanding the silence set L_i. However, a large L_i may degrade network performance by incurring unnecessary deferring of transmissions as follows: Let $E_i := L_i \backslash I_{r(i)}$. Then, while node i is transmitting, nodes in E_i will be unnecessarily deferring their transmission even though they do not interfere with node i. This phenomenon is called the *exposed node problem*. Thus, how to balance these two problems is a critical issue for improving the overall network performance. In the following, we derive an analytical relation for maximizing network capacity by balancing these two problems.

3.2. Characterization of CSMA MAC throughput

We focus on the behavior of an individual node and characterize its per-node throughput. In order to derive the throughput of a given node i, we need to find an explicit relation among the following three variables: the attempt probability that node i transmits in any virtual slot,[3] the conditional collision probability of node i given that a transmission attempt is made, and the virtual slot time of node i.

3.2.1. Derivation of attempt probability

The attempt probability under the exponential back-off mechanism has been derived in [8]. Specifically, the attempt probability τ_i that node i transmits in a randomly chosen virtual slot can be expressed as follows:

$$\tau_i = \frac{2(1-2q_i)}{(1-2q_i)(CW_m+1) + q_i CW_m(1-(2q_i)^m)}, \qquad (5)$$

where q_i is the conditional collision probability given that a transmission attempt is made, $m = \log_2\left(\frac{CW_M}{CW_m}\right)$ with CW_m and CW_M being the minimum and the maximum contention window sizes, respectively. Note that, if we assume an independent contention window size with an average value of $\overline{CW}_i$, τ_i in (5) is further simplified as $\tau_i = 2/(\overline{CW}_i + 1)$ [8].

3.2.2. Derivation of virtual slot time

The expressions for the conditional collision probability and the virtual slot will be different from those in [8]. We first derive the virtual slot time. Let v_i denote the expected virtual slot time. In order to characterize v_i, we need to model the channel behavior from the viewpoint of node i. By considering the channel status together with the node activities, we have a total of four channel states seen by node i.

- Successful transmission by node i: If node i receives an ACK frame within an interval of Short Interframe Space (SIFS) after the data frame is transmitted, it determines that the transmission is successful. Note that node i cannot detect whether or not transmissions by other nodes are successful, because a frame is determined to be successfully received if and only if the sender receives the corresponding ACK frame.
- Collision incurred by node i: If node i does not receive an ACK within an interval of SIFS after the data frame is transmitted, it determines that a collision occurs.
- Idle channel: If the received signal strength falls below the carrier sense threshold x_i, the channel is considered to be idle.
- Busy channel: If the received signal strength exceeds the carrier sense threshold x_i, the channel is considered to be busy. This busy channel results from transmissions of other nodes. Note that we do not distinguish whether this transmission is successful or not.

Let the average duration of each state be denoted by t_S, t_C, t_I, and t_B. Then, $t_S = T_H + T_P + T_{ACK} + SIFS + DIFS$, where T_H, T_P, and T_{ACK} are, respectively, the time required to transmit the header, the payload, and the acknowledgement. In addition, DIFS stands for the DCF Interframe Space [2]. Similarly, $t_C = T_H + T_P + SIFS + DIFS$, $t_I = \sigma$, where σ is the physical slot time, and t_B is approximated by $t_B = (t_S + t_c)/2$.[4]

Node i makes a transmission attempt with probability τ_i, and each attempt is successful with probability $1 - q_i$. Thus, the probability of successful transmission is $\tau_i(1-q_i)$. Also, the probability of collision is $\tau_i q_i$. The channel is idle when no node in $C_i \cup \{i\}$ transmits. Thus, the probability of idle channel is $(1-\tau_i)\prod_{j\in C_i}(1-\tau_j)$. Finally, the channel is busy when node i is in the back-off stage and at least one node in C_i transmits. Hence, the probability of busy channel is $(1-\tau_i)\left(1-\prod_{j\in C_i}(1-\tau_j)\right)$. By gathering everything together, the virtual slot time v_i can be expressed as

$$v_i = \tau_i(1-q_i)t_S + \tau_i q_i t_C + (1-\tau_i)\prod_{j\in C_i}(1-\tau_j)t_I + (1-\tau_i)\left(1-\prod_{j\in C_i}(1-\tau_j)\right)t_B. \qquad (6)$$

3.2.3. Derivation of conditional collision probability

What is left to be derived is the conditional collision probability q_i. The transmission of node i will result in collision if (i) at least one node in $I_{r(i)}$ transmits simultaneously at the beginning of the transmission of node i, or (ii) at least one node in H_i transmits before or during the transmission of node i. First, let η_i denote the probability of transmission attempt of node i. If we assume that η_j,

[3] We follow Bianchi's notion and define a virtual slot as the interval between the occurrences of two specific events. It may be much longer than the physical slot size σ.

[4] This approximation does not introduce significant error in our analysis since $t_S \approx t_c \gg t_I$.

$j \neq i$ is independent of transmission attempt of node i, then we have that the probability of node i finding no ongoing transmission in H_i^- becomes $\prod_{j \in H_i^-}(1-\eta_j)$.

Now, in order to account for the hidden nodes in H_i^+, let the vulnerable period, denoted by V, be defined as the time interval during which the transmission between node i and node $r(i)$ will fail if any node in H_i attempts to transmit. Then, $V = T_H + T_P$, where T_H and T_P are, respectively, the time required to transmit the header and the payload. Hence, the expected number of transmission attempt in duration of V is V/v_i. Now, the conditional collision probability q_i can then be expressed as

$$q_i = 1 - \prod_{j \in H_i^-}(1-\eta_j) \prod_{j \in I_{r(i)}}(1-\tau_j) \prod_{k \in H_i^+}(1-\tau_k)^{\frac{V}{v_k}}. \quad (7)$$

In (7), from the definition of τ_i, we further have $\eta_i \ll \tau_i$, which implies that the hidden node effect from H_i^+ dominates q_i over that from H_i^-. Hence, we have the following approximation for q_i.

$$q_i \approx 1 - \prod_{j \in I_{r(i)}}(1-\tau_j) \prod_{k \in H_i^+}(1-\tau_k)^{\frac{V}{v_k}}. \quad (8)$$

3.2.4. Derivation of per-node throughput

Let T_i denote the throughput of node i. Then, T_i can be expressed as

$$T_i = \frac{L\tau_i(1-q_i)}{v_i} = \frac{L\tau_i \prod_{j \in I_{r(i)}}(1-\tau_j)\prod_{k \in H_i^+}(1-\tau_k)^{\frac{V}{v_k}}}{v_i}, \quad (9)$$

where L is the payload size. Compared to the single-cell model in [8], the throughput model (9) is much more complicated in that physical carrier sense is considered through H_i^+ and v_i is included in the exponent. By a careful observation of (9), we can identify a tradeoff between the level of spatial reuse and the hidden node problem as follows: as the carrier sense threshold/transmit power is increased/decreased to allow more spatial reuse, the virtual slot time v_i will be decreased, which will increase T_i. Meanwhile, the hidden node problem becomes more severe, and the collision probability q_i will be increased, which in turn will decrease T_i. Consequently, there exists a tradeoff between the level of spatial reuse and the hidden node problem. It is of critical importance to quantify this tradeoff in order to find the optimal operating condition.

4. Throughput analysis

In this section, we derive optimal operating condition of the carrier sense threshold and the transmit power for maximizing network capacity.

4.1. Analysis in homogeneous networks

4.1.1. Interdependence among variables and differentiation of the node throughput

Under the assumption of a homogeneous network, we omit the node index i and use x and P to denote the carrier sense threshold and the transmit power, respectively. Furthermore, since $C_i = L_i$ by (3) and (4), let $X := (gP/x)^{\frac{1}{\theta}}$

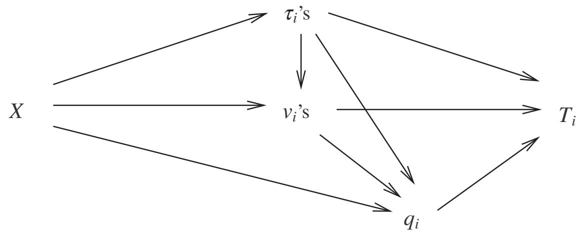


Fig. 1. Description of a logical interdependence among variables of node i on its throughput in a homogeneous network.

($= d_C(i,j) = d_L(i,j)$), which corresponds to the common carrier sense range for every node. From (6), (8) and (9), it can be easily shown that the node throughput T_i depends only on X. Thus, T_i is fully determined by P/x without need for considering x and P independently. Let T_s be the network capacity, which is expressed as

$$T_S = \sum_{i=i}^{N} T_i. \quad (10)$$

In order to find the optimal value of X for maximizing T_S in (10), we differentiate T_S with respect to X:

$$\frac{dT_S}{dX} = \sum_{i=i}^{N} \frac{dT_i}{dX}. \quad (11)$$

It turns out that it is not so simple to calculate the total derivative of T_S with respect to X in (11) due to the complicated interdependence among τ_i, q_i, v_i, and T_i. We tackle the problem by making use of the chain rule [24].

The first step for carrying out the operation is to specify the interdependence among τ_i, q_i, v_i, and T_i. After a careful observation on (5), (6), (8) and (9), we pick the interdependence as in Fig. 1.[5] From Fig. 1, let $\tau_i =: f_i(X)$, $v_i =: g_i(X, \tau_i, \{\tau_j\}_{j \in C_i})$, $q_i =: h_i(X, \{\tau_j\}_{j \in I_{r(i)}}, \{v_k\}_{k \in H_i^+})$, and $T_i =: F(\tau_i, v_i, q_i)$. The differentiation of T_i with respect to X can be expressed as

$$\begin{aligned}\frac{dT_i}{dX} &= \frac{\partial T_i}{\partial \tau_i}\frac{d\tau_i}{dX} + \frac{\partial T_i}{\partial v_i}\frac{dv_i}{dX} + \frac{\partial T_i}{\partial q_i}\frac{dq_i}{dX} \\ &= \frac{\partial F}{\partial \tau_i}\frac{df_i}{dX} + \frac{\partial F}{\partial v_i}\left(\frac{\partial g_i}{\partial X} + \frac{\partial g_i}{\partial \tau_i}\frac{df_i}{dX} + \sum_{j \in C_i}\frac{\partial g_i}{\partial \tau_j}\frac{df_j}{dX}\right) \\ &\quad + \frac{\partial F}{\partial q_i}\left[\frac{\partial h_i}{\partial X} + \sum_{j \in I_{r(i)}}\frac{\partial h_i}{\partial \tau_j}\frac{df_j}{dX} + \sum_{k \in H_i^+}\frac{\partial h_i}{\partial v_k}\left(\frac{\partial g_k}{\partial X} + \frac{\partial g_k}{\partial \tau_k}\frac{df_k}{dX}\right.\right. \\ &\quad \left.\left. + \sum_{l \in C_k}\frac{\partial g_k}{\partial \tau_l}\frac{df_l}{dX}\right)\right]. \end{aligned} \quad (12)$$

We have explicit formulas for h_i and F from (8) and (9), respectively. Thus, we can obtain all the terms related to

[5] It should be noted that the diagram in Fig. 1 is *not* intended to show the physical relation, but to introduce a logical relation for the mathematical purpose. Furthermore, it is *not the unique way* for describing the logical interdependence. In order to differentiate T_i with respect to X, we need to choose one specific interdependence among many possible alternatives. Dependencies that do not appear explicitly in Fig. 1 are considered implicit. The interested reader is referred to [24] for further detail.

h_i and F in (12). However, explicit expressions for f_i and g_i are still lacking. Consequently, we still have difficulty in deriving an explicit expression for dT_i/dX in (12). It should be noted that the difficulty *does not* come from the specific choice of the interdependence among many alternatives, but results from the *intrinsic nature* of the problem we are dealing with. As a matter of fact, for a given value of X, (5), (6) and (8) represent a nonlinear system for τ_i's, q_i, and υ_i's, which is extremely difficult to solve in an analytical manner. Even in the single-cell scenario where two unknowns τ_i and q_i constitute a nonlinear system, only a numerical solution based on fixed-point analysis is available in general [8]. In order to resolve this problem, we impose an assumption on the attempt probability τ_i. Let CW_i and $\overline{CW_i}$ denote the contention window size used by node i and its average, respectively. As we mentioned in the previous section, if we assume that $\overline{CW_i}$ is independent of X, the attempt probability τ_i is given as $\tau_i = 2/(\overline{CW_i}+1)$. Hence, (12) can be simplified as

$$\frac{dT_i}{dX} = \frac{\partial F}{\partial \upsilon_i}\frac{\partial g_i}{\partial X} + \frac{\partial F}{\partial q_i}\left[\frac{\partial h_i}{\partial X} + \sum_{k\in H_i^+}\frac{\partial h_i}{\partial \upsilon_k}\frac{\partial g_k}{\partial X}\right], \tag{13}$$

where $\upsilon_i = g_i(X)$, $q_i = h_i(X, \upsilon_i, \{\upsilon_j\}_{j\in H_i^+})$, $T_i = F(\upsilon_i, q_i)$. Now we are ready to derive an explicit relation between network capacity and system parameters.

4.1.2. Optimal condition for a homogeneous network with a symmetric topology

Consider a homogeneous network with a symmetric topology, in which $d_i = d$, $d_I(i,j) = d_I$, and $\tau_i = \tau$ for all $i,j \in \mathcal{N}$. Proposition 1 gives an approximate solution for the optimal carrier sense range, denoted by X^*, which maximizes network capacity.

Proposition 1. *For a homogeneous network with a symmetric topology, let $X^* = \arg\max_X T_S$. Also, let ρ denote the node density of the interference set $I_{r(i)}$, defined as $\rho = I_{r(i)}/(\pi d_I^2)$. If $\rho \neq 0$, then the optimal carrier sense rage X^* is*

$$X^* \approx \sqrt{(D+Q_\rho)^2 + R_\rho^2} - (Q_\rho + R_\rho),$$

where $D = d + d_I$, $Q_\rho = C_\rho/B_\rho$, $R_\rho = B_\rho/(2KA_\rho)$, and $A_\rho = \rho\left[(\pi-2\alpha)\kappa^2 + \frac{1}{2}\sqrt{4\kappa^2-1}\right]$, $B_\rho = 2\pi d_I(t_B - t_I)\ \log\left(\frac{1}{1-\tau}\right)\rho\ (1-\tau)^{\pi d_I^2\rho+1}$, $C_\rho = -B_\rho d_I + (1-\tau)[t_B - (t_B - t_I)\ (1-\tau)^{\pi d_I^2\rho}]$, $K = \tau V\log(1/(1-\tau))$, $\kappa = d_I/d = \beta^{\frac{1}{\theta}}$, $\alpha = \arccos(1/2\kappa)$. If $\rho = 0$, T_S is constant for all feasible $X \leqslant D$.

Proof. In a homogeneous network with a symmetric topology, T_i's are the same for $\forall i \in \mathcal{N}$. Thus, from (11), it is sufficient to find X^* such that $X^* = \arg\max_X T_i$. Let ρ denote the node density of the interference set $I_{r(i)}$, defined as $\rho = I_{r(i)}/(\pi d_I^2)$. Then, $I_{r(i)} = \pi d_I^2\rho = \pi\beta_i^{\frac{2}{\theta}} d^2\rho$. Under the assumption of a homogeneous network with a symmetric topology, together with the fact that $I_{r(i)}$ and τ are independent of X, (13) becomes

$$\frac{dT_i}{dX} = \frac{T_i}{\upsilon_i^2}\left[\tau V\log(1-\tau)\left(\frac{\partial H_i^+}{\partial X}\upsilon_i - H_i^+\frac{\partial \upsilon_i}{\partial X}\right) - \upsilon_i\frac{\partial \upsilon_i}{\partial X}\right]. \tag{14}$$

In (14), the number of hidden nodes H_i^+ is given as

$$H_i^+(X) = \begin{cases}\rho[-\phi X^2 + dX\sin\phi + \psi d_I^2], & \text{if } X \leqslant D;\\ 0, & \text{otherwise}.\end{cases} \tag{15}$$

where $\phi = \min\left[\pi, \arccos\left(\frac{X^2+d^2-d_I^2}{2dX}\right)\right]$, $\psi = \min\left[\pi, \pi - \arccos\left(\frac{d^2+d_I^2-X^2}{2dd_I}\right)\right]$, and $D = d + d_I$. Since there exists a constant ϵ such that T_i, $\upsilon_i \geqslant \epsilon > 0$ in the feasible region of X, in order to determine whether T_i is increasing or decreasing, from (14), it is sufficient to consider

$$f(X) = -K\left(\frac{\partial H_i^+}{\partial X}\upsilon_i - H_i^+\frac{\partial \upsilon_i}{\partial X}\right) - \upsilon_i\frac{\partial \upsilon_i}{\partial X},$$

where $K = -\tau V\log(1-\tau)$.

If $\rho = 0$, $f(X) = 0$ and T_i will be constant with respect to$X$. When $\rho \neq 0$, we consider the following two subcases.

(i) When $X > D$We have $H_i^+ = 0$ and $\partial H_i^+/\partial X = 0$ from (15). Also, it is straightforward to verify that $\partial\upsilon_i/\partial X \geqslant 0$ from (6). Thus, $f(X) = -\upsilon_i\partial\upsilon_i/\partial X \leqslant 0$ and T_i is a non-increasing function of X.

(ii) When $X \leqslant D$In order to get an explicit expression for X^*, we introduce the following approximations for H_i^+ and υ_i.

$$H_i^+(X) \approx \begin{cases}A_\rho(X-D)^2, & \text{if } X \leqslant D;\\ 0, & \text{otherwise},\end{cases}$$

where $A_\rho = H_i^+|_{R=d_I}/d^2 = \rho\left[(\pi-2\alpha)\kappa^2 + \frac{1}{2}\sqrt{4\kappa^2-1}\right]$, $\alpha = \arccos(1/2\kappa)$, $\kappa = d_I/d$. Also, $\upsilon_i(X) \approx B_\rho X + C_\rho$, where

$$\begin{aligned}B_\rho &= \frac{d\upsilon_i}{dX}|_{X=d_I}\\ &= -\tau(t_S - t_C)\frac{dq_i}{dX}|_{X=d_I} + 2\pi d_I(t_B - t_I)\log\left(\frac{1}{1-\tau}\right)\\ &\quad \rho(1-\tau)^{\pi d_I^2\rho+1},\end{aligned}$$

and $C_\rho = -B_\rho d_I + \upsilon_i|_{X=d_I}$. By further introducing

$$\frac{dq_i}{dX}|_{X=d_I} \approx -2d\log\left(\frac{1}{1-\tau}\right)A_\rho(1-\tau)^{I_{r(i)}+\frac{VH_i^+|_{X=d_I}}{\upsilon_i|_{X=d_I}}},$$

we have

$$\begin{aligned}B_\rho &\approx 2\log\left(\frac{1}{1-\tau}\right)(1-\tau)^{\pi d_I^2\rho}\\ &\quad\times\left[\pi d_I(t_B - t_I)(1-\tau)\rho + d\tau(t_S - t_C)A_\rho(1-\tau)^{\frac{V}{\upsilon_I}d^2A_\rho}\right]\\ &\approx 2\pi d_I(t_B - t_I)\log\left(\frac{1}{1-\tau}\right)\rho(1-\tau)^{\pi d_I^2\rho+1},\end{aligned}$$

and $C_\rho = -B_\rho d_I + \upsilon_I$, where

$$\begin{aligned}\upsilon_I &\approx \tau_i\left[\left(1-\frac{1}{\kappa}\right)t_S + \frac{1}{\kappa}t_B\right] + (1-\tau)t_B - (t_B - t_I)(1-\tau)^{\pi d_I^2\rho+1}\\ &\approx (1-\tau)[t_B - (t_B - t_I)(1-\tau)^{\pi d_I^2\rho}].\end{aligned}$$

Now, with omitting the subscript ρ for simplicity,

$$f(X) \approx -ABKX^2 - (2ACK + B^2)X - BC + AKD(BD + 2C).$$

The discriminant of $f(X)$ is $\Delta = 4A^2(BD + C)^2K^2 + B^4 > 0$. Thus, $f(X) = 0$ has two distinct real roots, denoted by $X^{*,1}$ and $X^{*,2}$ ($X^{*,1} > X^{*,2}$). Then,

$$X^{*,1}, X^{*,2} = \frac{\pm\sqrt{\Delta} - (2ACK + B^2)}{2ABK}.$$

Since $\frac{df}{dX_i}\Big|_{X=X^{*,1}} < 0$ and $\frac{df}{dX_i}\Big|_{X=X^{*,2}} > 0$, the maximum of T_i is attained at $X^{*,1}$. From (i) and (ii), $X^* = X^{*,1}$ when $\rho \neq 0$. □

Corollary 1 gives how X^* in Proposition 1 changes with respect to the node density ρ.

Corollary 1. *For a homogeneous network with a symmetric topology, there exists* $\underline{\rho}$ *such that* X^* *is an increasing function of the node density* ρ *if* $\rho \geqslant \underline{\rho}$. *Furthermore, as the node density increases, the optimal carrier sense range* X^* *converges to* D (=$d + d_I$), *which covers the entire interference range, i.e.,* $\lim_{\rho\uparrow\infty} X^* = D(=d + d_I)$.

Proof. Let $f(\rho) := D + Q_\rho$ and $g(\rho) := R_\rho$. Then,

$$\frac{\partial X^*}{\partial \rho} = -\left[1 - \frac{f}{\sqrt{f^2+g^2}}\right]\frac{df}{d\rho} - \left[1 - \frac{g}{\sqrt{f^2+g^2}}\right]\frac{dg}{d\rho}.$$

From Proposition 1, we have $\lim_{\rho\uparrow\infty} Q_\rho = \infty$ and $\lim_{\rho\uparrow\infty} R_\rho = 0$. Hence, $\lim_{\rho\uparrow\infty} f(\rho) = \infty$ and $\lim_{\rho\uparrow\infty} g(\rho) = 0$. Consequently, $1 - f/\sqrt{f^2+g^2} = 1 - 1/\sqrt{1+(g/f)^2} \downarrow 0$ as $\rho \to \infty$. In a similar manner, $1 - g/\sqrt{f^2+g^2} = 1 - (g/f)/\sqrt{1+(g/f)^2} \uparrow 1$ as $\rho \to \infty$. Thus, together with the fact that $df/d\rho > 0$ and $dg/d\rho < 0$, there exists $\underline{\rho}$ such that $\frac{\partial X^*}{\partial \rho} > 0$ for $\rho \geqslant \underline{\rho}$. Furthermore, it is straightforward from Proposition 1 that $\lim_{\rho\uparrow\infty} X^* = D$ because $\lim_{\rho\uparrow\infty} Q_\rho = \infty$ and $\lim_{\rho\uparrow\infty} R_\rho = 0$. □

Fig. 2 depicts, in a qualitative manner, the relation between the optimal carrier sense range X^* and the node density ρ. When the node density is not so high, X^* will be X_1^*, which is smaller than $d + d_I$. As ρ increases, X^* will get close to $d + d_I$ as X_2^*. It should be noted that $X = d + d_I$ corresponds to the case of exactly covering the entire interference range, thus eliminating all the hidden nodes. Also, note that X^* can become negative for small values of ρ from Proposition 1. In practice, the carrier sense threshold should be smaller than the receive threshold, which implies that X should be at least larger than d. The practical meaning of X^* smaller than d in Proposition 1 is that there are virtually no interfering neighbor nodes and we can reduce the carrier sense range to the minimum feasible value, without much concern about the interference or the hidden node problem.

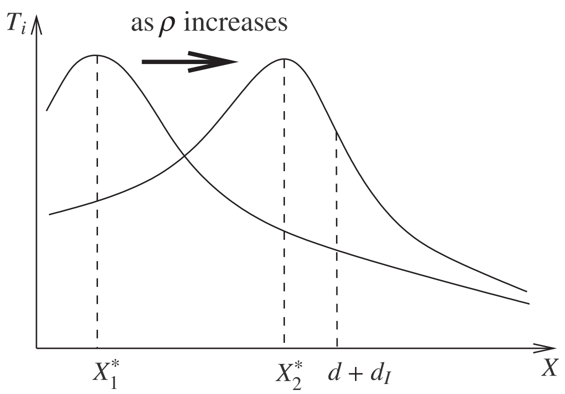


Fig. 2. The optimal carrier sense range X^* as a function of the node density ρ.

4.1.3. Optimal condition for a homogeneous network with a random topology

Now we consider a homogeneous network with a random topology in Proposition 2.

Proposition 2. *In a homogeneous network with a random topology, let* $X_i^* = \arg\max_X T_i$ *and* $\rho_i = I_{r(i)}/(\pi d_I^2)$ *for node i. If* $\rho_i \neq 0$, $X_i^* \approx \sqrt{(D+Q_\rho)^2 + R_\rho^2} - (Q_\rho + R_\rho)$, *where all the constants are the same as those in Proposition 1, except* d_i *instead of d. If* $\rho_i = 0$, T_i *is constant for all feasible* $X_i \leqslant D$.

Proof. For tractability, assume that $v_k \approx v_i$ for $k \in H_i^+$ (which holds true if $\rho_i \approx \rho_j$ for $j \in C_i$.) Then, the rest of the proof will follow the same line of Proposition 1. □

By Proposition 2, we know that $X_i^{*\prime}$s are different among nodes because d_i's and ρ_i's are different. Thus, *no single value* of X can maximize the throughput of every node at the same time. However, if information on d_i's and ρ_i's is available, one simple approximation of $X^* = \arg\max_X \sum_{i\in N} T_i$ is $X^* \approx \frac{1}{N}\sum_{i\in N} X_i^*$ from Proposition 2.

4.2. Nonhomogeneous network

We consider a nonhomogeneous network where each system parameter can be arbitrarily adjusted for each node. In a nonhomogeneous network, x_i's and P_i's should be considered as independent parameters in order to properly describe T_i. The problem of maximizing network capacity can be formulated as follows.

$$\max_{\mathbf{x},\mathbf{P}} \left[T_S \equiv \sum_{i\in N} T_i(\mathbf{x},\mathbf{P})\right], \quad (16)$$

where $\mathbf{x} = (x_1, \ldots, x_N)$ and $\mathbf{P} = (P_1, \ldots, P_N)$. Since the search space in (16) contains that of a homogeneous network (which has a restriction of $x_1 = \cdots = x_N$ and $P_1 = \cdots = P_N$), it is obvious that network capacity of a nonhomogeneous network can be at least not smaller than that of a homogeneous network, if properly tuned. A critical issue in practice is how to design an algorithm for tuning $\mathbf{x}$ and $\mathbf{P}$ in a distributed manner. We show that any distributed algorithm that maximizes the throughput of each node without coordinating with other nodes will fail to maximize the overall network capacity. Intuitively, the throughput of each node will increase as its carrier sense threshold/transmit power increases given that those of other nodes are fixed. Hence, without consideration of other nodes, each node will increase its carrier sense threshold and transmit power as much as possible, which will significantly degrade the overall network performance. Proposition 3 validates this intuition.

Proposition 3. *Any distributed algorithm that maximizes the throughput of each node by tuning its own carrier sense threshold and transmit power without coordination with other nodes will fail to maximize network capacity.*

Proof. If node i increases x_i with $\mathbf{x}_{-i}$ and $\mathbf{P}$ fixed where $\mathbf{x}_{-i} = (x_1,\ldots,x_{i-1},x_{i+1},\ldots,x_N)$, then v_i will be decreased because C_i is reduced in (6), with q_i unchanged. Thus, T_i will be increased. In a similar manner, if node i increases P_i with $\mathbf{x}$ and $\mathbf{P}_{-i}$ fixed where $\mathbf{P}_{-i} = (P_1,\ldots,P_{i-1},P_{i+1},\ldots,P_N)$, then q_i will be decreased because H_i^+ is reduced in (8), with v_i unchanged. Thus, T_i will be increased. Accordingly,

$$\frac{\partial T_i}{\partial x_i} \geqslant 0 \quad \text{and} \quad \frac{\partial T_i}{\partial P_i} \geqslant 0. \tag{17}$$

Any distributed algorithm in which every node maximizes its own throughput without coordinating with other nodes can be formulated as

$$\max_{x_i,P_i} T_i(\mathbf{x},\mathbf{P}), \quad \text{for} \quad i \in N. \tag{18}$$

From (17), the optimal solution of (18) is $(x_i,P_i) = (\bar{x}_i,\bar{P}_i)$ where $\bar{x}_i$ and $\bar{P}_i$ are maximum feasible values of x_i and P_i. Thus, $\bar{\mathbf{y}} = ((\bar{x}_1,\bar{P}_1),\ldots,(\bar{x}_N,\bar{P}_N))$ is the equilibrium of (18). □

From Proposition 3, if every node works in a selfish manner, the operating point will converge to the trivial equilibrium that is obviously not optimal from the system viewpoint. This phenomenon results from the fact that T_i not only depends on x_i and P_i, but also depends on $\mathbf{x}_{-i}$ and $\mathbf{P}_{-i}$. Although Proposition 3 is intuitive, its implication on the design of a distributed algorithm is quite significant. In fact, the problem of maximizing network capacity in a fully distributed manner corresponds to a non-cooperative game [12]. Consequently, each node i not only need to consider its own throughput T_i (as profit), but also need to introduce a certain penalty G_i (as price) for its adverse impact on other nodes. In this manner, (18) should be modified as

$$\max_{x_i,P_i} [T_i(\mathbf{x},\mathbf{P}) - G_i(x_i,P_i)], \quad \text{for} \quad i \in N, \tag{19}$$

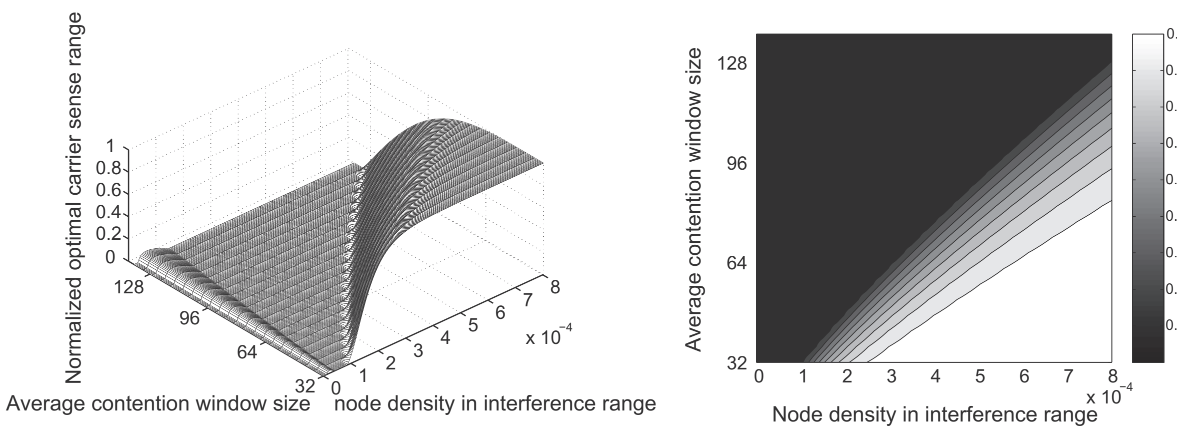
where G_i is a pricing function of node i, which is non-decreasing and convex in x_i and P_i. In fact, one approach for solving (19) has recently been proposed in [16]. It will be an interesting avenue of future work to compare the performance of different kinds of a pricing function G_i.

4.3. Discussion on remaining issues

Now we point out several important issues which are worthy of further investigation. First, we have not considered data rate adjustment according to the signal quality (such as the auto-rate function available in most IEEE 802.11a/b/g chipsets). There are 4 data rates (1, 2, 5.5, 11 Mb/s) available in 802.11b and 8 data rates (6, 9, 12, 18, 24, 36, 48, 54 Mb/s) available in 802.11a/g. Usually the higher the SINR value, the higher the data rate at which the transmission can sustain. For a given value of SINR, one may then choose the highest possible data rate (which allows correct decoding for that given SINR value) in order to maximize system throughput. Since a higher data rate can be sustained with a larger SINR threshold, the data rate sustained is a function of the carrier sense threshold and the transmit power. This issue has been studied in [3] without consideration of the MAC specifics. It will be interesting to investigate the impact of multiple data rates on network MAC throughput. Second, the effect of the RTS/CTS handshake mechanism has not been considered in our analysis. We believe that the RTS/CTS mechanism has a minimal impact on our derivation, and likely it will only reduce the duration of collision state t_C. A more thorough investigation is underway to validate our intuition.

5. Simulation study

In this section, we carry out a simulation study to validate the derived relation between network capacity and



(a) Normalized optimal carrier sense range

(b) Contour plot of normalized optimal carrier sense range

Fig. 3. Optimal carrier sense range as a function of node density and contention window size.

system parameters. In particular, we corroborate the model presented in Section 3.2 as well as Propositions 1 and 2.

First, we look into the relation between network capacity and system parameters such as the contention window size and the node density. Fig. 3 shows the numerical result on the normalized optimal carrier sense range, i.e., $(X^* - d)/d_I$ as a function of the node density ρ and the average contention window size $\overline{CW}$ when $d = 100$ m and $\kappa = 3$. Note that only non-negative values of $(X^* - d)/d_I$ are shown in Fig. 3 by plotting $\max((X^* - d)/d_I, 0)$. For a fixed value of $\overline{CW}$, we can identify from Fig. 3 that $(X^* - d)/d_I$ increases (except when $\rho \approx 0$) and converges to one as ρ increases. For a given value of ρ, $(X^* - d)/d_I$ decreases as $\overline{CW}$ increases. This behavior is due to the fact that the attempt probability τ decreases with $\overline{CW}$. As a result, the effective node density also decreases. Fig. 3 shows that $(X^* - d)/d_I$ becomes negative for large values of $\overline{W}$, which implies that there are no nearby interfering nodes.

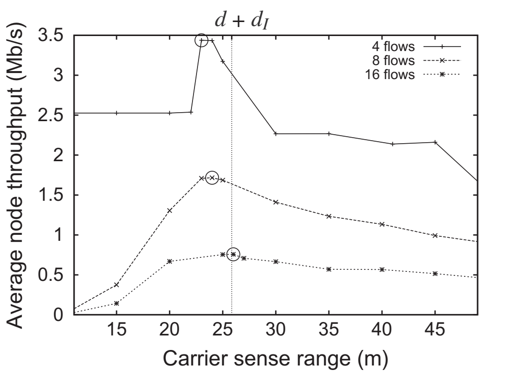
Now, we further study how the node density and the SINR threshold impact on the optimal carrier sense range via ns-2 simulations. The simulation study is carried out by using the 802.11 Ext model [25] newly included in ns-2 since version 2.33. First, we adopt a symmetric circular topology, in which there are two concentric circles with a radius of 15 m and 25 m. N senders and N receivers are evenly located, respectively, on the outer and inner circle. The corresponding receiver to each sender is on the same diameter, and thus the distance between a sender and a receiver, d, is 10 m. The parameter values used in the simulation study are listed in Table 1.

Table 1
Parameters used in ns-2 simulations.

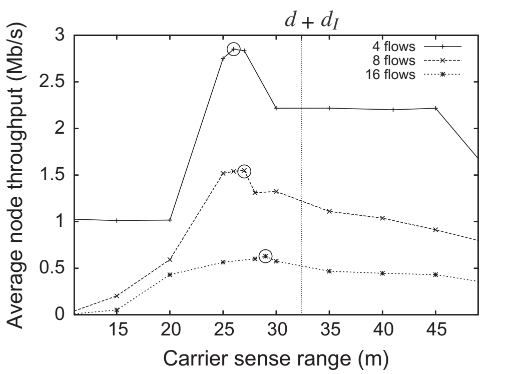
Propagation	Two-ray	RTS/CTS	Disabled
CW size	15 slots	Noise floor	−96 dBm
Data rate	(6,12,24) Mb/s	SINR thresh	(2.5,5.0,15.8) dB

Simulations are performed for $N = 4, 8, 16$ with the data rate $r = 6, 12, 24$ Mb/s in Fig. 4. The maximum point in each curve is marked with a circle. The dotted vertical lines in Fig. 4a–c, which are 25.8 m, 32.4 m, and 49.8 m, respectively, denote the points of $X = d + d_I$, which covers the entire interference range. In the meantime, our analysis gives the following optimal values, respectively for each figure in Fig. 4: (8.20, 8.09, 15.04), (11.28, 19.33, 30.96), (42.32, 49.63, 49.81), and (15.04, 30.96, 49.81). The analytical results become more accurate as the node density increases, which can be also inferred from Fig. 3a. In addition, the region of small non-zero values when the node density is near zero in Fig. 3a, which corresponds to the approximation error, is responsible for the decreasing trend of the analytical values for $N = 4$.

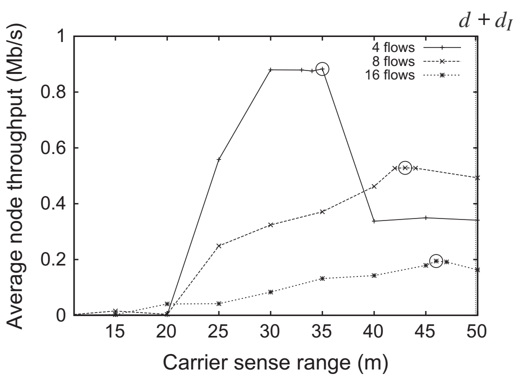
Here, several observation can be made. First, the optimal carrier sense range is smaller than $d + d_I$ in general. Furthermore, as the node density increases, the optimal



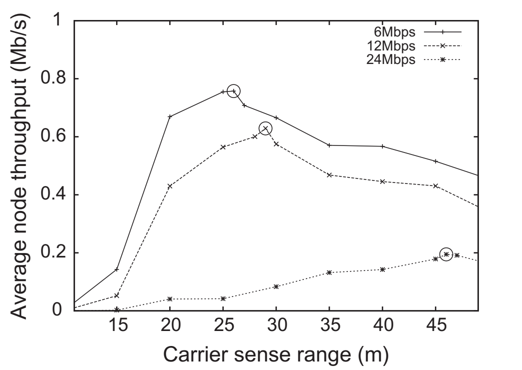
(a) Node throughput vs. carrier sense range when r = 6 Mb/s



(b) Node throughput vs. carrier sense range when r = 12 Mb/s

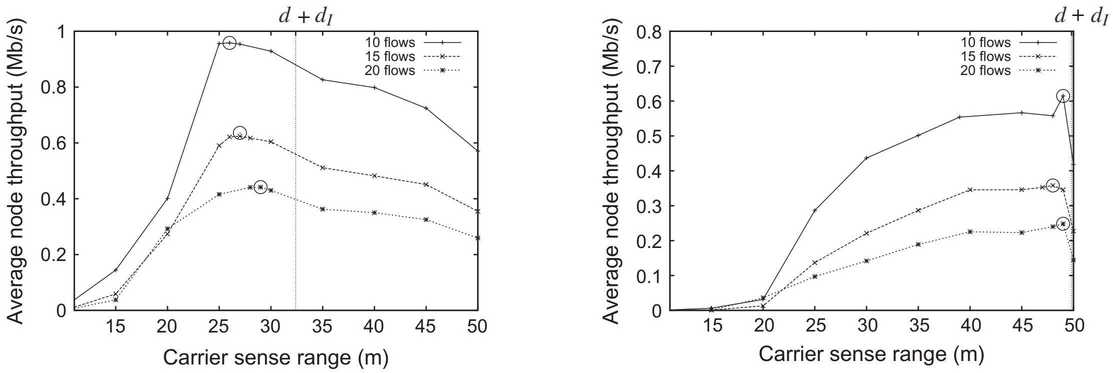


(c) Node throughput vs. carrier sense range when r = 24 Mb/s



(d) Node throughput vs. carrier sense range when N = 16

Fig. 4. Average node throughput as a function of the carrier sense range under a circular topology.



(a) Node throughput vs. carrier sense range when r = 12 Mb/s (b) Node throughput vs. carrier sense range when r = 24 Mb/s

Fig. 5. Average node throughput as a function of the carrier sense range under a random topology.

point converges to $d + d_I$, which verifies the result in Corollary 1. We further show in Fig. 4d how the node throughput changes with respect to change in the data rate, which validates that the interference range expands as the data rate increases because of increase in the SINR threshold.

Now, as a more realistic scenario, we consider the following random topology. In the circular area with the radius of 25 m, total of N transmission pairs are randomly located with d = 10 m. Fig. 5 shows the average node throughput as a function of the carrier sense range for various data rates. The analytical values obtained from Proposition 1 for Fig. 5a and b are, respectively, (24.39, 30.47, 31.95) and (49.78, 49.81, 49.81). It can be verified in Fig. 5 that these analytical values match the trend of simulation results.

Overall, we can summarize that, instead of covering the entire interference range, it is beneficial to increase the carrier sense threshold, or equivalently, decrease the carrier sense range. In particular, the typical value of the carrier sense threshold used in the IEEE 802.11 WLAN may be too conservative for network capacity and need to be carefully examined.

6. Conclusion and future work

In this paper, we have investigated the issue of maximizing network capacity of CSMA wireless networks. In particular, we have explicitly incorporated the carrier sense threshold and the transmit power into the analysis. In a homogeneous network, we have found that the optimal carrier sense range is smaller than the value for exactly covering the entire interference range. In a nonhomogeneous network, the problem of maximizing network capacity in a fully distributed manner has been shown to be a noncooperative game.

We have identified several future research avenues. First, our model can be extended to incorporate the effects of multiple data rates and the RTS/CTS handshake mechanism. Furthermore, based on the insight shed from the analysis, an efficient distributed algorithm can be developed for tuning the carrier sense threshold and the transmit power. One key step for designing such distributed algorithms is to properly define the pricing function in (19). Since there have been extensive studies on power control in the context of topology maintenance, we may leverage existing power control algorithms to develop a framework for joint control of the transmit power and the carrier sense threshold.

References

[1] J.C. Hou, K.-J. Park, T.-S. Kim, L.-C. Kung, Medium access control and routing protocols for wireless mesh network, in: E. Hossain, K.K. Leung (Eds.), Wireless Mesh Networks: Architectures and Protocols, Springer, 2007. October.
[2] IEEE, Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications, IEEE Standard 802.11, 1999.
[3] T.-S. Kim, H. Lim, J.C. Hou, Improving spatial reuse through tuning transmit power, carrier sense threshold, and data rate in multihop wireless networks, in: Proceedings of ACM MobiCom, 2006.
[4] X. Yang, N.H. Vaidya, On the physical carrier sense in wireless ad hoc networks, in: Proceedings of IEEE INFOCOM, 2005.
[5] H. Zhai, Y. Fang, Physical carrier sensing and spatial reuse in multirate and multihop wireless ad hoc networks, in: Proceedings of IEEE INFOCOM, 2006.
[6] J. Zhu, X. Guo, L. Yang, W.S. Conner, Leveraging spatial reuse in 802.11 mesh networks with enhanced physical carrier sensing, in: Proceedings of IEEE ICC, 2004.
[7] J. Zhu, X. Guo, L. Yang, W.S. Conner, S. Roy, M.M. Hazra, Adapting physical carrier sensing to maximize spatial reuse in 802.11 mesh networks, Wiley Wireless Communications and Mobile Computing 4 (2004) 933–946.
[8] G. Bianchi, Performance analysis of the IEEE 802.11 distributed coordination function, IEEE Journal on Selected Areas in Communications 18 (3) (2000).
[9] F. Calì, M. Conti, E. Gregori, Dynamic tuning of the IEEE 802.11 protocol to achieve a theoretical throughput limit, IEEE/ACM Transactions on Networking 8 (6) (2000) 785–799.
[10] A. Kumar, E. Altman, D. Miorandi, M. Goyal, New insights from a fixed point analysis of single cell IEEE 802.11 WLANs, in: Proceedings of IEEE INFOCOM, 2005.
[11] K. Medepalli, F.A. Tobagi, Towards performance modeling of IEEE 802.11 based wireless networks: a unified framework and its applications, in: Proceedings of IEEE INFOCOM, 2006.
[12] T. Başar, G.J. Olsder, Dynamic noncooperative game theory, SIAM Series in Classics in Applied Mathematics (1999). January.
[13] J. Zhu, B. Metzler, X. Guo, Y. Liu, Adaptive CSMA for scalable network capacity in high-density WLAN: a hardware prototyping approach, in: Proceedings of IEEE INFOCOM, 2006.
[14] Y. Zhu, Q. Zhang, Z. Niu, J. Zhu, On optimal physical carrier sensing: theoretical analysis and protocol, in: Proceedings of IEEE INFOCOM Miniconferences, 2007.
[15] H. Ma, R. Vijayakumar, S. Roy, J. Zhu, Optimizing 802.11 wireless mesh networks based on physical carrier sensing, IEEE/ACM Transactions on Networking 17 (5) (2009) 1550–1563.

[16] K.-J. Park, J.C. Hou, T. Başar, H. Kim, Noncooperative carrier sense game in wireless networks, IEEE Transactions on Wireless Communications 8 (1) (2009) 5280–5289.
[17] N. Li, J.C. Hou, L. Sha, Design and analysis of a MST-based distributed topology control algorithm for wireless ad-hoc networks, IEEE Transactions on Wireless Communications 4 (3) (2005) 1195–1207.
[18] R. Ramanathan, R. Rosales-Hain, Topology control of multihop wireless networks using transmit power adjustment, in: Proceedings of IEEE INFOCOM, 2000.
[19] R. Wattenhofer, L. Li, P. Bahl, Y.-M. Wang, Distributed topology control for power efficient operation in multihop wireless ad hoc networks, in: Proceedings of IEEE INFOCOM, 2001.
[20] K.-J. Park, L. Kim, J.C. Hou, Adaptive physical carrier sense in topology-controlled wireless networks, IEEE Transactions on Mobile Computing 9 (1) (2010) 87–97.
[21] J. Monks, V. Bharghavan, W.-M. Hwu, A power controlled multiple access protocol for wireless packet networks, in: Proceedings of IEEE INFOCOM, 2001.
[22] H. Ma, J. Zhu, S. Roy, S.Y. Shin, Joint transmit power and physical carrier sensing adaptation based on loss differentiation for high density IEEE 802.11 WLAN, Computer Networks 52 (9) (2008) 1703–1720.
[23] P. Gupta, P.R. Kumar, The capacity of wireless networks, IEEE Transactions on Information Theory 46 (2) (2000) 388–404.
[24] R.C. Buck, Advanced Calculus, Waveland Press Inc., 2003.
[25] Q. Chen, F. Schmidt-Eisenlohr, D. Jiang, M. Torrent-Moreno, L. Delgrossi, H. Hartenstein, Overhaul of IEEE 802.11 modeling and simulation in ns-2, in: Proceedings of ACM MSWiM, 2007.

Kyung-Joon Park received his B.S., M.S., and Ph.D. degrees all from the School of Electrical Engineering and Computer Science (EECS), Seoul National University (SNU), Seoul, Korea in 1998, 2000, and 2005, respectively. He is currently a research assistant professor in the School of EECS at SNU. He has been a post-doctoral research associate in the Department of Computer Science, University of Illinois at Urbana-Champaign (UIUC) from 2006 to 2010. He worked for Samsung Electronics, Suwon, Korea as a senior engineer in 2005–2006, and was a visiting graduate student, supported by the Brain Korea 21 Program, in the Department of Electrical and Computer Engineering at UIUC in 2001–2002. He has current research interests in characterization and design of medical-grade protocols for wireless healthcare systems, analysis of malicious and selfish behavior for wireless network security, and design and analysis of self-adjusting protocols for wireless environments. He is the Gold Prize Winner of the 4th Inside Edge International Thesis Competition from Samsung Electro-Mechanics in 2008. He has received a Distinguished Paper Prize at the OPNET Conference in 2005. He is also a winner of the Human-Tech Thesis Prize from Samsung Electronics in 2003, 2004, and 2005.

Jihyuk Choi received the B.S. and M.S. degrees from Seoul National University, Seoul, Korea in 1998 and 2000, respectively. He is a Ph.D. candidate at the Department of Electrical and Computer Engineering of the University of Illinois at Urbana-Champaign (UIUC). He was a research engineer at Electronics and Telecommunications Research Institute (ETRI), Korea from 2000 to 2003. He was also a senior researcher at LG Electronics Institute of Technology, Korea from 2003 to 2006. His research interests include wireless network protocols and network security.

Jennifer C. Hou was born on September 26, 1964 in Taipei, Taiwan. She received her B.S.E. degree in Electrical Engineering from National Taiwan University, Taiwan, ROC in 1987, M.S.E degrees in Electrical Engineering and Computer Science (EECS) and in Industrial and Operations Engineering (I & OE) from the University of Michigan, Ann Arbor, MI in 1989 and in 1991, and Ph.D. degree in EECS also from the University of Michigan, Ann Arbor, MI in 1993. She was an assistant professor in Electrical and Computer Engineering at the University of Wisconsin, Madison, WI from 1993 to 1996, and an assistant/associate professor in Electrical Engineering at Ohio State University, Columbus, OH from 1996 to 2001. She joined the University of Illinois Computer Science faculty in 2001. She was a principal researcher in networked systems and served as the director of the Illinois Network Design and Experimentation (INDEX) research group. She has supervised several federally and industry funded projects in the areas of network modeling and simulation, network measurement and diagnostics, and both the theoretical and protocol design aspects of wireless sensor networks. She has published (with her former advisor, students, and colleagues) over 160 papers in archived journals, book chapters, and peer-reviewed conferences. Her work on topology control and performance limits in wireless networks has been widely cited. Dr. Hou has been involved in organizing several international conferences sponsored by professional organizations such as ACM Mobicom, IEEE INFOCOM, IEEE MASS, and IEEE RTAS, as well as editor in archival journals and magazines such as IEEE Trans. on Computers, IEEE Trans. on Wireless Communications, IEEE Trans. on Mobile Computing, IEEE Trans. on Parallel and Distributed Systems, IEEE Wireless Communication Magazine, Elsevier Computer Networks, and ACM Trans. on Sensor Networks. Dr. Hou was a recipient of an ACM Recognition of Service Award in 2004 and 2007, a Cisco University Research Award from Cisco, Inc., 2002, a Lumley Research Award from Ohio State University in 2001, a NSF CAREER award from the Network and Communications Research Infrastructure, National Science Foundation in 1996-2000 and a Women in Science Initiative Award from The University of Wisconsin-Madison in 1993–1995. She was elected as an IEEE Fellow and an ACM Distinguished Scientist in 2007. Dr. Hou passed away on December 2, 2007 in Houston, Texas at the age of 43.

Yih-Chun Hu received his B.S. Magna Cum Laude in 1997 in Computer Science and Mathematics from the University of Washington. He received his Ph.D. in 2003 from the Computer Science Department at Carnegie Mellon University. He was a Postdoctoral Researcher at the University of California, Berkeley. He is currently an assistant professor in the Department of Electrical and Computer Engineering at the University of Illinois at Urbana-Champaign. His general research interests are in security and systems, with emphasis on the areas of secure systems and mobile communications. He has published papers in the areas of secure Internet routing, DDoS-resilient forwarding, secure routing in wireless ad hoc networks, security and anonymity in peer-to-peer networks, efficient cryptographic mechanisms for routing security, and the design and evaluation of multi-hop wireless network routing protocols, including Quality-of-Service mechanisms for ad hoc networks.

Hyuk Lim received his B.S., M.S., and Ph.D. degrees all from the School of Electrical Engineering and Computer Science, Seoul National University, Seoul, Korea in 1996, 1998, and 2003, respectively. He is currently an assistant professor in the Department of Information and Communications, and the Department of Nanobio Materials and Electronics, Gwangju Institute of Science and Technology (GIST), Gwangju, Republic of Korea. He was a postdoctoral research associate in the Department of Computer Science, University of Illinois at Urbana-Champaign in 2003–2006. His research interests include analytical modeling and empirical evaluation of computer networking systems, network protocol design and performance analysis for wireless networks, measurement and diagnostics for wired/wireless networks, and location-aware applications in ubiquitous sensor networks.